0. introduction

"Note that neither the current set of representative glyphs in the standard nor the glyphs from many commonly available non-mathematical fonts achieve the ideals set forth here."

These ruminations are based principally on UTR 25 (Unicode and Mathematics), revision 9 — section 2.11 (Geometric Shapes) in particular. The quotations in italics at the start of some sections are taken from the same document.

This is a follow-up to comments on UTR 25, version 7, dated 2006/Jan/26, but no knowledge of that earlier document is required.

Code points and character names from Unicode 5.0 are augmented, where necessary, by those shown for block 2B00~2BFF, Miscellaneous Symbols and Arrows, described in WG2/N3264 – Summary of Repertoire for FPDAM 4, dated 27-Apr-2007.

Proper sizing and naming of glyphs for shapes is largely a matter of constraint satisfaction, not unlike the monthly logic puzzle magazines, but there is a subjective element. Subjective content is highlighted in blue: in all other instances, anybody not happy with a result will have to fault the preceding logic, or change the rules.

The conclusions are —

(i) that, consistency being the *sine qua omnes* of all mathematical theories (see Appendix), we should aim for total consistency;

(ii) that the position of a sample glyph in the Table should be consistent with any sizing information provided in the character name, or its associated notes;

(iii) where no other sizing information is available, the Shapes Table should provide an unambiguous guide to the relative sizes of geometric shapes;

(iv) that any changes in the attributes of glyphs within the Shapes Table should only be approved after due consideration to the use of said glyphs in composite glyphs elsewhere in the standard.

1. consistent use of names

> *"The intended sizes of existing characters and their names in Unicode as shown in the code charts are not always consistent."*

The table shown overleaf is a (more or less) faithful copy of Table 2.5, from §2.11 of version 9 of UTR 25.

- Glyphs with no coloured background are not germane to this discussion.
- Glyphs with a green background have a size defined by the character name (or, in the case of U+2022 and U+2981, given in the associated notes), and appear in the appropriate column of Table 2.5.
- Glyphs with a grey background have no size information in the character name, or anywhere else in the comments, and could conceivably appear in any column of the table.
- Glyphs with a red background have a size defined by the character name, but that size conflicts with the column in which the glyphs appear in Table 2.5.

Firstly, this consistency is undesirable: font designer A may size his glyphs according to the character name, while font designer B sizes his according to the column name. A typographer lays out text using font A, but later switches to font B, only to find that, in addition to some subtle changes in the plain text, the formulae have undergone some visually unacceptable changes in the sizes of the operators.

Secondly, there is no need for it:

(i) accept the character name, or, failing that, the notes, as a normative description of the intended size of the corresponding glyphs

(ii) show the representative glyph in the appropriate column of the Shapes Table, and

(iii) if a character of the required size and shape is not shown in the Table, a new one can be defined, with sizing information in either the character name or the comments.

shape table Mk I

size	tiny	very small		small		medium small		medium (default1)		regular (default2)		large	
triangle left				◂ 25c2	◃ 25c3					◀ 25c0	◁ 25c1		
triangle right				▸ 25b8 =2023	▹ 25b9					▶ 25b6	▷ 25b7		
triangle up				▴ 25b4	▵ 25b5					▲ 25b2	△ 25b3		
triangle down				▾ 25be	▿ 25bf					▼ 25bc	▽ 25bd		
square		⬝ 2b1d	⬞ 2b1e	▪ 25aa	▫ 25ab	◾ 25fe	◽ 25fd	◼ 25fc	◻ 25fb	■ 25a0	□ 25a1	⬛ 2b1b	⬜ 2b1c
diamond				⬩ 2b29	⋄ 22c4	⬥ 2b25	⬦ 2b26			◆ 25c6	◇ 25c7		
lozenge				⬪ 2b2a	⬫ 2b2b	⬧ 2b27	⬨ 2b28			⧫ 29eb	◊ 25ca		
pentagon right										⭓ 2b53	⭔ 2b54		
pentagon up										⬟ 2b1f	⬠ 2b20		
hexagon horizontal										⬣ 2b23	⎔ 2394		
hexagon vertical										⬢ 2b22	⬡ 2b21		
arabic star				⭑ 2b51 →066d	⭒ 2b52	⋆ 22c6	⭐ 2b50	★ 2605	☆ 2606				
ellipse horizontal										⬬ 2b2c	⬭ 2b2d		
ellipse vertical										⬮ 2b2e	⬯ 2b2f		
circle	⋅ 22c5 →00b7	∙ 2219 →00b7	∘ 2218	• 2022	◦ 25e6 →2022	⦁ 2981	⚬ 26ac	⚫ 26ab	⚪ 26aa	● 25cf	○ 25cb	⬤ 2b24	◯ 25ef
circled circles	⊙ 2299	☉ 2609			⌾ 233e								
circled circles	⨀ 2a00	⦿ 29bf	⊚ 229a		⦾ 29be	◉ 25c9	◎ 25ce						

2. consistency of visual impact

> *"… shapes of the same size should ideally have roughly the same visual "impact" as opposed to the same nominal height or width. The shapes shown here for a given size all have the same area."*

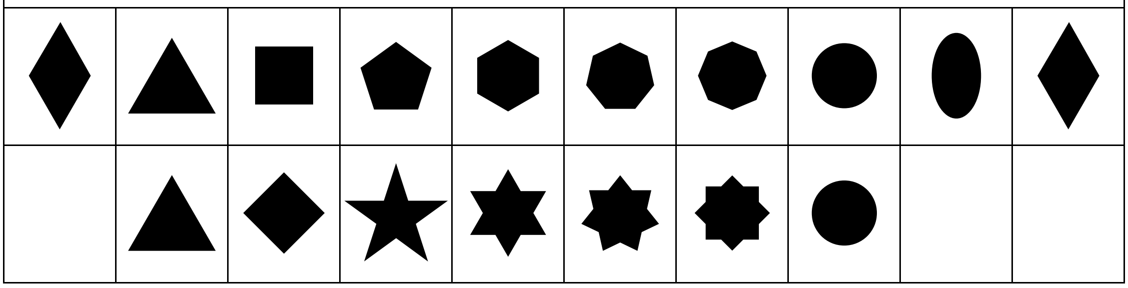
The second of these statements is untrue, in the context of UTR 25. It was true in my earlier document, where it referred to the table presented in that document, but that table is not the one currently appearing in UTR 25.

It's time to do what should have been done earlier: allow for visual impact in the proposed table of shapes appearing in this document, instead of leaving the matter open.

Unfortunately, there is no immediately obvious objective measure of visual impact. Using the area of the bounding box, the diamond, for instance, would be deemed to have twice the impact of a square of equal area, which is unlikely to match most people's experience.

The diameter of the escribed circle turns out to be equally unpromising—for one thing, the square and the diamond have precisely the same escribed circle.

prototype shapes, with equal areas of black



Testing was done at 36pt, with higher-order polygons and 2-parameter stars included to give a feel for any progression that may be present.

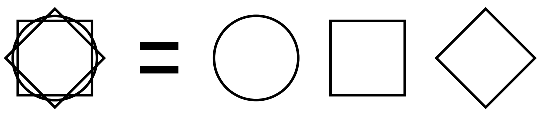
In the event, no progression was detected, but the following shapes appear, to one observer at least, to have more or less equal impact.

regular shapes, with subjectively equal visual impact									
90%	88%	100%	100%	100%	100%	100%	100%	100%	90%
⧫ 29eb	▲ 25b2	■ 25a0	⬟ 2b1f	⬢ 2b22			● 25cf	⬮ 2b2e	⧫ 29eb
	88%	95%	82%	90%	34pt	35pt	100%		
	▲ 25b2	◆ 25c6	★ 2605				● 25cf		

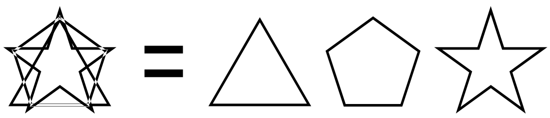
The major and minor axes of the ellipse are in the same ratio to each other as the major and minor diagonals of the lozenge. Similar rules could apply to

25AC horizontal black rectangle,
25AD horizontal white rectangle,
25AE vertical black rectangle,
25AF vertical white rectangle,
25B0 horizontal black parallelogram and
25B1 horizontal white parallelogram.

The basic outlines look OK together



and even the awkward squad look acceptable.



3. a consistent progression of sizes

> *"The suggested sizes here correspond to a geometric progression where for each size all characters have the same visual impact."*

This is a rather awkward sentence. It is actually the scaling from "prototype" to "regular" that aims to guarantee equal visual impact. The progression of sizes from "tiny" to "large" is a separate issue.

The ideographic uses of the regular squares:

25A0 BLACK SQUARE
= moding mark (in ideographic text)

25A1 WHITE SQUARE
* may be used to represent a missing ideograph

fixes the sizes of these glyphs: probably sitting on the baseline, vertically centred, aligned on the same axis as the plus sign and, therefore, extending to about caps height. And this in turn decides the height of the regular-sized circles.

We need another fixed point, and our progression can be defined. By a happy circumstance, it was found that setting the radius of U+22C5 DOT OPERATOR to the stem width, and using a geometric (rather than an arithmetic) progression, the circles thus defined met the other requirements:

— the nested circles showed sufficient white between the outlines to be identifiable, and

— "medium" circles have diameters roughly equal to the x-height, and the white circle will enclose most mathematical operators comfortably, e.g:

⊕ U+2295 CIRCLED PLUS

— "regular" sized circles are roughly equal in height to the fg-height, and the white circle will enclose most mathematical operators with a little white space showing, which is convenient for N-ary operations, e.g:

⨁ U+2A01 N-ARY CIRCLED PLUS OPERATOR

— U+25EF LARGE CIRCLE is large enough to enclose uppercase M.

This solution is not unique. In fact, all sizes could be set by eye, but when that has been done, the resultant sizes will (give or take a few percent) fit a geometric progression. The STIXGeneral circles, for example, increase in diameter by approximately 30% at each step from "very small" to "large", without precisely fitting a GP.

If the other "regular" shapes are scaled using the same geometric series, then we might reasonably expect the shapes in each column of the resultant table to show the same visual impact.

But surely this is most unlikely to be true unless the regular shapes have already been adjusted to show the same visual impact? And that does not appear to be the case with Table 2.5 of UTR 25. The square and the diamond, in particular, look out of kilter.

The measure of visual impact is, of course, still subjective. If the font designer's boss says the Arabic star loses its impact at small sizes faster than the other shapes, who are we to gainsay him?

On the brighter side, most of these shapes can be generated automatically within FontLab. While the choice of scale factor from prototype to regular is 100% subjective, applying that factor is another automatic process, so experimentation is easy.

Scaling to other sizes can also be done automatically and default Type1 hinting is usually adequate for printing purposes, so that the addition of existing shapes at new sizes places no real load on the font designer, beyond the tedious business of TrueType hinting for low-res screens.

Maybe the quotation at the start of this section could be rephrased thus:

> *It was found that a set of circles whose radii increased according to a simple geometric progression met the many individual and collective requirements for circular glyphs.*
>
> *The sample glyphs shown here for other shapes have been scaled from the corresponding regular-sized glyphs, using exactly the same geometric progression, in the expectation that all shapes of a given size would thus have very similar visual impact.*

4. consistent use of characters

Once upon a time, mathematicians would choose the symbols they wanted, and a typeface would be constructed (or, more probably, extended) to provide the required symbols.

Likewise with mathematical software: MathCAD and Mathematica provide their own fonts.

Nowadays, the mathematician can plunder the Unicode set, confident(?) that a font incorporating those symbols will have the appropriate appearance.

Likewise with software: Fortress, which aims to accept 2-D notation, can specify its character set in terms of Unicode values.

In the gap between these two states lie two formal notations, conceived independently of Unicode, which subsequently redefined their symbol sets in terms of Unicode values, *viz*. APL and Z.

The APL Character Repertoire, adopted in Berlin in 2000, can be seen, pro tem, at http://www.dkuug.dk/jtc1/sc22/open/n3067.pdf.

A "current" working draft of the Z standard can be found at http://www.mcs.vuw.ac.nz/courses/COMP426/2006T1/2002/Documents/Z-standard.ps . Dated 24th August 2000, pp18-24 list the code values appropriate to each character/symbol used in Z.

Both of these documents predate the appearance of the first version of UTR 25 on 10th October 2001.

Mathematical symbols vary in size from towering multi-storied integrals down to complete invisibility. Infix operators, though, are less varied in size, mostly slim verticals or something having much the same bounding box as the "plus" sign. This means medium-sized shapes can conveniently be used as operators. (Or rather, they could be, if there were provision for them within the standard.)

Mathematicians have their differences regarding preferred sizes of glyphs (which may just be a case of settling for what is available), but in APL and Z we have a printed record of the preferred appearance of these notations—an appearance which should not be affected by changes within the Unicode standard.

The following examples are therefore drawn from APL and Z, as well as general mathematics.

S◁R	Z, domain restriction
S▷R	Z, range restriction
A△B	set theory, symmetric difference
▽ƒ	analysis, the Hamilton operator applied to *ƒ*
A∘.×B	APL, outer product
1○Theta	APL, *sin*(θ)
f○g	category theory and Z, composition of functions

Unfortunately, all these examples are flawed.

5. triangles

S◁R	Z, domain restriction
S▷R	Z, range restriction
▽ƒ	analysis, the Hamilton operator applied to ƒ

Z notation uses medium-sized triangles for its restrictions. As currently formulated, Table 2.5 of UTR 25 shows large triangles only, albeit with the let-out clause

> "the default size shown in the code charts would be in the column marked 'regular', while for many font implementations, a size corresponding to the column marked 'medium' is chosen".

http://en.wikipedia.org/wiki/Relational_algebra uses a medium-sized triangle to denote "antijoin".

"Complete Calculi for Matrices" (available at enormous expense from http://portal.acm.org/citation.cfm?id=362274) uses similar-sized triangles as selection operators.

The notes to U+2206 suggest its use for symmetric difference. http://en.wikipedia.org/wiki/Symmetric_difference uses a delta character, showing contrast.

http://mathworld.wolfram.com/SymmetricDifference.html says[1],

> "The symmetric difference of sets A and B is variously written as A⊖B, A▽B, A+B (Borowski and Borwein 1991) or A△B (Harris and Stocker 1998, p. 3). All but the first notation should probably be deprecated since each of the other symbols has a common meaning in other areas of mathematics."

The Hamilton operator is typically a regular-sized symbol. A small, but not necessarily unrepresentative, sample—the first half dozen analysis books to hand—shows it sitting on the baseline, with a degree of stress and contrast consistent with the letters of the mathematical font in use.

http://www.w3.org/TR/REC-MathML/chap6/ISOTECHe1.html recommends the use of U+2207 for the Hamilton operator. And so does Unicode, which says

> 2207 NABLA
> = gradient, del

[1] retyped – the triangles in the online version are neither so large nor equilateral

Since the Hamiltonian operator is no more nor less than the gradient function, this note could be amended to read

> 2207 NABLA
> = gradient (or Hamiltonian), del

and the corresponding entry deleted from

> 25BD WHITE DOWN-POINTING TRIANGLE
> = Hamilton operator .

U+2207, like its offspring the Laplace operator (U+2206), is best regarded as a letter, not a shape.

With the Hamilton operator thus taken care of elsewhere, and the contradiction removed, we can now standardise on medium-sized triangles for infix operators (other than symmetric differences, perhaps):

size	tiny	very small		small		medium small		medium (default1)		regular (default2)		large	
triangle up				▴ 25b4	▵ 25b5			▲ 25b2	△ 25b3				

and similarly for other orientations.

6. diamonds, lozenges, stars

The published entries for these shapes are just plain wrong.

size	tiny	very small		small		medium small		medium (default1)		regular (default2)		large	
diamond				⬩ 2b29	⋄ 22c4	⬥ 2b25	⬦ 2b26			◆ 25c6	◇ 25c7		
lozenge				⬪ 2b2a	⬫ 2b2b	⬧ 2b27	⬨ 2b28			⧫ 29eb	◊ 25ca		
arabic star				⭑ 2b51 →066d	⭒ 2b52	⋆ 22c6	⭐ 2b50	★ 2605	☆ 2606				

The proposed character names for U+2B25~2B28 are

> BLACK MEDIUM DIAMOND
> WHITE MEDIUM DIAMOND
> BLACK MEDIUM LOZENGE
> WHITE MEDIUM LOZENGE.

If the glyphs are intended to be medium-small, they ought to be (ought to have been) named thus:

> BLACK MEDIUM-SMALL DIAMOND
> WHITE MEDIUM-SMALL DIAMOND
> BLACK MEDIUM-SMALL LOZENGE
> WHITE MEDIUM-SMALL LOZENGE.

This would have the merit of leaving the font designer the option of providing medium- or regular-sized glyphs for U+25C6, 25C7, 29EB and 25CA.

Alternatively, because the above line of action may now be denied to us, the glyphs should be shown in the MEDIUM column:

size	tiny	very small		small		medium small		medium (default1)		regular (default2)		large	
diamond				⬩ 2b29	⋄ 22c4			⬥ 2b25	⬦ 2b26	◆ 25c6	◇ 25c7		
lozenge				⬪ 2b2a	⬫ 2b2b			⬧ 2b27	⬨ 2b28	⧫ 29eb	◊ 25ca		

The diamond used in APL could equally well be either a medium-small or a medium sized glyph: the APL Character Repertoire specified U+22C4, which is now shown as a small glyph, although there is no sizing information in the code charts, and none proposed.

The proposed character name for U+2B50 is WHITE MEDIUM STAR.

The simplest correction would be to rename the character
> WHITE MEDIUM-SMALL STAR

before it is too late.

If it is already too late, the table should be amended as follows:

size	tiny	very small		small		medium small		medium (default1)		regular (default2)		large	
arabic star				⭑ 2b51 →066d	⭒ 2b52	⋆ 22c6			⭐ 2b50	★ 2605	☆ 2606		

which has the merit of consistency, and completeness can be satisfied later (version 5.2, say) by adding the characters

> WHITE MEDIUM-SMALL STAR
> BLACK MEDIUM STAR.

U+22C6 has no sizing information, but it is specifically the APL "star operator", and should continue to have the size expected for that character, in an APL context. A good idea of what that looks like can be gleaned from http://www.aplusdev.org/keyboard.html or http://www.wickensonline.co.uk/apl/union-large.png.

7. circles

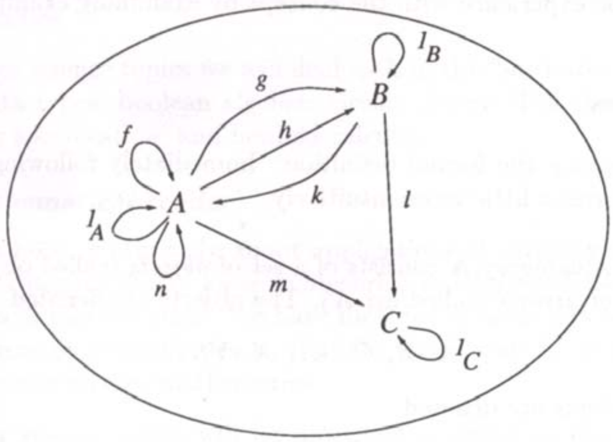
A∘.×B	APL, outer product
1○Theta	APL, *sin*(θ)
f∘g	category theory, composition of functions

Peter J Cameron, *Sets, Logic and Categories*, says (p10):

> Two functions $f : X \to Y$ and $g : Y \to Z$ can be *composed* to give a function $f \circ g : X \to Z$ defined by
>
> $$f \circ g = \{(x, z) : (\exists y \in Y)((x, y) \in f \text{ and } (y, z) \in g)\}.$$
>
> (Note that $(f \circ g)(x) = g(f(x))$: this unfortunate reversal arises because of our notation for functions, writing the function on the left of its argument.)

R F C Walters, *Categories and Computer Science*, says (p4):

> **Remark.** Informally, a category is an algebra of functions between sets, and hence looks something like this:



> The principal operation in a category is composition. Hence for example, given $g : A \to B$ and $l : B \to C$ we can form $l \circ g : A \to C$; in the example above $l \circ g$ must be m since m is the only arrow from A to C.

J M Spivey, *The Z Notation, A Reference Manual*, (http://spivey.oriel.ox.ac.uk/mike/zrm/zrm.pdf) comes down firmly on both sides of the fence (p97):

Name

id – Identity relation
⨾ – Relational composition
∘ – Backward relational composition

Definition

$\mathrm{id}\, X == \{ x : X \bullet x \mapsto x \}$

$[X, Y, Z]$

$\_ ⨾ \_ : (X \leftrightarrow Y) \times (Y \leftrightarrow Z) \rightarrow (X \leftrightarrow Z)$
$\_ \circ \_ : (Y \leftrightarrow Z) \times (X \leftrightarrow Y) \rightarrow (X \leftrightarrow Z)$

$\forall R : X \leftrightarrow Y;\ S : Y \leftrightarrow Z \bullet$
$R ⨾ S = S \circ R = \{ x : X;\ y : Y;\ z : Z \mid (x \mapsto y) \in R \land (y \mapsto z) \in S \bullet x \mapsto z \}$

Description

The *identity relation* id X on a set X relates each member of X to itself. The *composition* $R ⨾ S$ of two relations $R : X \leftrightarrow Y$ and $S : Y \leftrightarrow Z$ relates a member x of X to a member z of Z if and only if there is at least one element y of Y to which x is related by R and which is itself related to z by S. The notation $S \circ R$ is an alternative to $R ⨾ S$.

Laws

$(x \mapsto x') \in \mathrm{id}\, X \Leftrightarrow x = x' \in X$
$(x \mapsto z) \in R ⨾ S \Leftrightarrow (\exists y : Y \bullet (x \mapsto y) \in R \land (y \mapsto z) \in S)$

$R ⨾ (S ⨾ T) = (R ⨾ S) ⨾ T$
$\mathrm{id}\, X ⨾ R = R$
$R ⨾ \mathrm{id}\, Y = R$

$\mathrm{id}\, V ⨾ \mathrm{id}\, W = \mathrm{id}(V \cap W)$

$(f \circ g)(x) = f(g(x))$

They may differ on semantics, but there appears to be common agreement on the expected size of the glyph: something a little larger than the upper loop of the *g*.

The following examples use successively U+2218, U+25E6 and U+26AC, with medium math spaces either side:

f∙g f⚬g f∘g

and, considerations of style, weight and contrast aside, it is the third example that appears closest in size to common usage. Not, as currently suggested in the notes, U+2218.

Of course, one could design a font where U+2218 was of a size appropriate for denoting functional composition, but that font would not be able to provide a full set of graduated circles meeting the explicit and implicit requirements of UTR 25.

The alternative is to amend the notes:

> 26AC MEDIUM SMALL WHITE CIRCLE
> = composite function

and delete the corresponding entry from

> 2218 RING OPERATOR
> = composite function.

Or tell the mathematicians to get used to smaller circles.

8. composites

U+233E APL FUNCTIONAL SYMBOL CIRCLE JOT is something of a misfit. It was once a composite of the APL circle and the APL jot, which would have been U+25CB WHITE CIRCLE for the APL circle, and U+2218 RING OPERATOR for the APL jot.

As shown, U+233E is precisely the glyph the APLers had in mind when choosing their codepoints, and is composed of the “circle” and “jot” glyphs they had in mind.

Nowadays, however, the WHITE CIRCLE is bigger and the RING OPERATOR is smaller, and things look different. The glyph shown in UTR 25 for U+233E is no longer a composite of those two glyphs, but is instead is a composite of U+26AA MEDIUM WHITE CIRCLE and U+25E6 WHITE BULLET.

It is not clear what should happen here but, briefly, APLers can stick with vintage 2000 glyphs or vintage 2000 codepoints, but not both[2]. The circle-jot glyph would then, presumably, be the composite of the chosen “circle” and “jot”.

If a picture is worth a thousand words, maybe a table can save a few hundred.

[2] if they wish to use a Unicode font which complies with UTR 25 recommendations

vintage	expected			actual		
2000	∘ 2218	○ 25cb	⌾ 233e	∘ 2218	○ 25cb	⌾ 233e
2007	∘ 2218	○ 25cb	⌾ 233e	∘ 2218	○ 25cb	⌾ 233e
2008?	∘ 2218	○ 25cb	⊚ 229a	∘ 2218	○ 25cb	⊚ 229a
2008.alt	◦ 25e6	⚪ 26aa	⌾ 233e	◦ 25e6	⚪ 26aa	⌾ 233e

This is one example of side effects on composites caused by changes in the constituent parts. There are others: anti-restriction in Z, for instance, would be similarly affected by changes in the domain and range restriction symbols in Z.

9. a counter-example

One symbol unaffected by changes in the sizes of shapes is U+2395 APL FUNCTIONAL SYMBOL QUAD — and the sixteen composites based on it.

The APL quad, like the Hamiltonian, like the Laplace operator, is more a letter than a shape, and this perhaps points the way to a solution: separate the geometric shapes from their current semantics, and define abstract shapes and mathematical symbols separately.

Shapes would then have as much or as little meaning as dingbats, and the circles used for function composition, or binary operators in group theory, or set comprehension, or contour integrals, would have their sizes defined by the font designer (or the font’s sponsor), rather than by Unicode.org .

Similar rules already apply to arrows, which do not have associated semantics defined, in the majority of cases.

10. other uses of shapes

Mathematical notation uses shapes for conventional signs, and on an *ad hoc* basis. The symbol for function composition is almost always a circle, for instance, while the symbol for the binary operator in a group can be chosen more or less arbitrarily. In the latter case, medium and medium-small shapes seem to be the most popular, and it would be nice if both these sizes could be provided for the simpler shapes (triangles, squares, diamonds, lozenges, circles and stars).

Mathematical notation is not the only area to use geometric shapes, of course. Some other areas of application are:

as exemplars, as in “this is a hexagon ⬡ ”;

> Exemplars can vary in size, colour and orientation.

with other, more application specific, symbols in technical diagrams;

> One corollary of this is that shapes should be drawn *without* optical “correction”. The accuracy of a diagram is assessed by straight edge and compass, ruler and protractor – not judgement by an eye accustomed to seeing things out of kilter.

as bullets;

> So long as “small” shapes are acceptable to the typographer, the world is well provided for in this area.

for interpunction;

> A few visits to churches and cemeteries will quickly show that size and shape are quite varied in this area. As well as the usual shapes (typically straight-edged shapes, when cut in stone), a kind of slanting mid tilde appears to be popular (locally, at least). It is not clear, however, whether these are “shapes” in the sense of this document, or substitutes for the space character(s).

11. larger issues

The proper way to finish a report such as this is with a set of substantiated recommendations and conclusions. Sorry.

The recommendations given on Page 1 relate mostly to UTR 25, being guidance on how a mathematical font meeting various criteria might be constructed, and how UTR 25 should show the glyphs thus produced.

But those difficulties may also be symptoms of larger issues arising as much from holes in policy as from anything else.

Q1: Would APL have been better protected if it had been a registered character repertoire? Or a registered encoding?

> (Bear in mind here that it is the additional level of detail regarding the *glyph* which produces the undesired effect.)

Q2: Is there any precise way to discover which glyphs are considered "composite", and what other glyphs they are deemed to built up from?

> (N.B: this is not the same thing as being a composite character.)

Is there any way to check whether changes to glyph A will have side-effects on glyph B, because B is a composite of which A is a part?

Q3: What would be the advantages and disadvantages of declaring shapes to be abstract shapes only, and semantically void?

One thing, however, is beyond question[3]:

—if a character name describes a shape as "medium"-sized, the glyph should be "medium"-sized, of a size consonant with other "medium"-sized glyphs, and UTR 25 should show that glyph in Table 2.5 in the column headed "medium".

12. some shape tables

These tables are not necessarily 100% correct. The hex code may have been mistyped; the font may not be the most recent version; Word may have substituted a glyph from another font; Acrobat may display the tables using a different version of the specified font. For all that, they give a good overview of the current state of the art of implementing shapes in large fonts.

[3] IMHO

shape table Mk II

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
triangle left				◂ 25c2	◃ 25c3			◀ 25c0	◁ 25c1	◀	◁		
triangle right				▸ 25b8 =2023	▹ 25b9			▶ 25b6	▷ 25b7	▶	▷		
triangle up				▴ 25b4	▵ 25b5			▲ 25b2	△ 25b3	▲	△		
triangle down				▾ 25be	▿ 25bf			▼ 25bc	▽ 25bd	▼	▽		
square		⬝ 2b1d	⬞ 2b1e	▪ 25aa	▫ 25ab	◾ 25fe	◽ 25fd	◼ 25fc	◻ 25fb	■ 25a0	□ 25a1	⬛ 2b1b	⬜ 2b1c
diamond				⬩ 2b29	⋄ 22c4			⬥ 2b25	⬦ 2b26	◆ 25c6	◇ 25c7		
lozenge				⬪ 2b2a	⬫ 2b2b			⬧ 2b27	⬨ 2b28	⧫ 29eb	◊ 25ca		
horizontal rectangle										▬ 25ac	▭ 25ad		
vertical rectangle										▮ 25ae	▯ 25af		
parallel-ogram										▰ 25b0	▱ 25b1		
pentagon right										⭓ 2b53	⭔ 2b54		
pentagon up										⬟ 2b1f	⬠ 2b20		
hexagon horizontal										⬣ 2b23	⎔ 2394		
hexagon vertical										⬢ 2b22	⬡ 2b21		

shape table Mk II, contd.

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
arabic star				⭑ 2b51 →066d	⭒ 2b52	⋆ 22c6	☆	★	⭐ 2b50	★ 2605	☆ 2606		
star of David								✡	✡	✡	✡		
circle	⋅ 22c5	∙ 2219 (00b7)	∘ 2218	• 2022	◦ 25e6	⦁ 2981	⚬ 26ac	⚫ 26ab	⚪ 26aa	● 25cf	○ 25cb	⬤ 2b24	◯ 25ef
circled circles (default1)	⊙ 2299	☉ 2609			⌾ 233e								
circled circles (default2)	⨀ 2a00	⦿ 29bf	⊚ 229a	◉	⦾ 29be	◉ 25c9	◎ 25ce	◉	◎				
ellipse horizontal										⬬ 2b2c	⬭ 2b2d		
ellipse vertical										⬮ 2b2e	⬯ 2b2f		

Cambria & Cambria Math v1.01

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
triangle left				25c2	25c3			25c0	25c1				
triangle right				25b8 =2023	25b9			25b6	25b7				
triangle up				25b4	25b5			25b2	△ 25b3				
triangle down				25be	25bf			25bc	25bd				
square		2b1d	2b1e	25aa	25ab	25fe	25fd	25fc	25fb	■ 25a0	□ 25a1	2b1b	2b1c
diamond				2b29	⋄ 22c4			2b25	2b26	25c6	25c7		
lozenge				2b2a	2b2b			2b27	2b28	29eb	◊ 25ca		
horizontal rectangle										25ac	▭ 25ad		
vertical rectangle										25ae	25af		
parallel-ogram										25b0	25b1		
pentagon right										2b53	2b54		
pentagon up										2b1f	2b20		
hexagon horizontal										2b23	2394		
hexagon vertical										2b22	2b21		

Cambria & Cambria Math v1.01, contd.

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
arabic star				2b51 →066d	2b52	★ 22c6			2b50	2605	2606		
star of David													
circle	· 22c5	• 2219 (00b7)	∘ 2218	● 2022	25e6	2981	26ac	26ab	26aa	25cf	25cb	2b24	25ef
circled circles (default1)	⊙ 2299	2609			233e								
circled circles (default2)	⨀ 2a00	29bf	⊚ 229a		29be	25c9	25ce						
ellipse horizontal										2b2c	2b2d		
ellipse vertical										2b2e	2b2f		

MS Mincho v2.31

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
triangle left				◂ 25c2	◃ 25c3			◀ 25c0	◁ 25c1				
triangle right				▸ 25b8 =2023	▹ 25b9			▶ 25b6	▷ 25b7				
triangle up				▴ 25b4	▵ 25b5					▲ 25b2	△ 25b3		
triangle down				▾ 25be	▿ 25bf					▼ 25bc	▽ 25bd		
square		2b1d	2b1e	▪ 25aa	▫ 25ab	25fe	25fd	25fc	25fb	■ 25a0	□ 25a1	2b1b	2b1c
diamond				2b29	⋄ 22c4			2b25	2b26	◆ 25c6	◇ 25c7		
lozenge				2b2a	2b2b			2b27	2b28	29eb	◊ 25ca		
horizontal rectangle										▬ 25ac	▭ 25ad		
vertical rectangle										▮ 25ae	▯ 25af		
parallel-ogram										▰ 25b0	▱ 25b1		
pentagon right										2b53	2b54		
pentagon up										2b1f	2b20		
hexagon horizontal										2b23	2394		
hexagon vertical										2b22	2b21		

MS Mincho v2.31, contd.

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
arabic star				2b51 →066d	2b52	⋆ 22c6			2b50	★ 2605	☆ 2606		
star of David													
circle	⋅ 22c5	∙ 2219 (00b7)	∘ 2218	• 2022	◦ 25e6	2981	26ac	26ab	26aa	● 25cf	○ 25cb	2b24	◯ 25ef
circled circles (default1)	⊙ 2299	☉ 2609			233e								
circled circles (default2)	2a00	29bf	⊚ 229a		29be	◉ 25c9	◎ 25ce						
ellipse horizontal										2b2c	2b2d		
ellipse vertical										2b2e	2b2f		

STIX General v0.9

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
triangle left				25c2	25c3					25c0	25c1		
triangle right				25b8 =2023	25b9					25b6	25b7		
triangle up				25b4	25b5					25b2	25b3		
triangle down				25be	25bf					25bc	25bd		
square		2b1d	2b1e	25aa	25ab	25fe	25fd	25fc	25fb	25a0	25a1	2b1b	2b1c
diamond				2b29	22c4			2b25	2b26	25c6	25c7		
lozenge				2b2a	2b2b			2b27	2b28	29eb	25ca		
horizontal rectangle										25ac	25ad		
vertical rectangle										25ae	25af		
parallel-ogram										25b0	25b1		
pentagon right										2b53	2b54		
pentagon up										2b1f	2b20		
hexagon horizontal										2b23	2394		
hexagon vertical										2b22	2b21		

STIX General v0.9, contd.

size	tiny	very small		small (bullet)		medium small		medium (default1)		regular (default2)		large	
arabic star				⭑ 2b51 →066d	⭒ 2b52	⋆ 22c6			⭐ 2b50	★ 2605	☆ 2606		
star of David													
circle	⋅ 22c5	∙ 2219 (00b7)	∘ 2218	• 2022	◦ 25e6	⦁ 2981	⚬ 26ac	⚫ 26ab	⚪ 26aa	● 25cf	○ 25cb	⬤ 2b24	◯ 25ef
circled circles (default1)	⊙ 2299	☉ 2609			233e								
circled circles (default2)	⨀ 2a00	⦿ 29bf	⊚ 229a		⦾ 29be	◉ 25c9	◎ 25ce						
ellipse horizontal										⬬ 2b2c	⬭ 2b2d		
ellipse vertical										⬮ 2b2e	⬯ 2b2f		

Appendix: consistency

from "*Introduction to Logic*", by Patrick Suppes:

COROLLARY 1.15. *The system L is consistent, i.e., there is no* wf $\mathscr{A}$ *such that both* $\mathscr{A}$ *and* $\sim\mathscr{A}$ *are theorems of L.*

PROOF. By Proposition 1.11, every theorem of L is a tautology. The negation of a tautology cannot be a tautology, and, therefore, it is impossible for both $\mathscr{A}$ and $\sim\mathscr{A}$ to be theorems of L.

Notice that L is consistent if and only if not all wfs of L are theorems. For, clearly, if L is consistent, then there are wfs which are not theorems (e.g., the negations of theorems). On the other hand, by Lemma 1.10(c), $\vdash_L \sim\mathscr{A} \supset (\mathscr{A} \supset \mathscr{B})$, and so, if L were inconsistent, i.e., if some wf $\mathscr{A}$ and its negation $\sim\mathscr{A}$ were provable, then, by MP, any wf $\mathscr{B}$ would be provable. (This equivalence holds for any theory having modus ponens as a rule of inference and in which Lemma 1.10(c) is provable.) A theory in which not all wfs are theorems is often said to be *absolutely consistent*, and this definition is applicable even to theories not containing a negation sign.

from "*Introduction to Mathematical Logic*" by Elliott Mendelson:

In many cases, it is not easy to decide if a set of premises is consistent simply by "looking" at them, and consequently it is desirable to develop an analytical technique for investigating consistency. To begin with, two sentences are said to be *contradictory* if one is the negation of the other; a *contradiction* is a conjunction of two contradictory sentences, that is, it is a conjunction of the form S & –S. Now it is easy to see that a set of premises is inconsistent if a contradiction can be logically derived, for if the premises could all be true together, we could construct an example violating Criterion I—that is, true premises and the necessarily false conclusion S & –S. Our technique for investigating the consistency of a set of premises is thus to attempt to derive a contradiction. We approach the problem of attempting to derive a contradiction in the same general way we approach the problem of deriving a given conclusion. The essential difference is that in deriving a given conclusion the terminal point of the derivation is fixed in advance, whereas in deriving a contradiction, the terminal point is *any* contradiction—it does not matter what particular one.

Not the best examples, perhaps, but briefly and informally, within a formal system or mathematical theory, certain propositions are provable. If P is one of these provable propositions, and it subsequently transpires that ~P (not-P) is also provable, then the contradiction, (P & ~P), is also provable. This contradiction can then be used to prove any and every proposition you can construct within that formal system, or that mathematical theory*.

It is then no longer possible to distinguish Truth from Falsehood because nothing is false – and the formal system is useless.

Similarly with rule systems: admit one contradiction, and you admit them all.

* if the system permits Modus Ponens as a valid inference rule